Answer Explanations for: ACT Form 0964E from *Preparing for the ACT*

Mathematics

1) D) Remember how absolute values work. First, do any computation within the absolute value. If the result is positive or zero, keep it as it is. If the result is negative, make it positive. The initial equation simplifies to $|4| - |-4|$ = 4 – 4 = 0. See absolute values.

2) G) Turn this situation into an equation. When modeling a linear situation, the y-intercept (b in y = mx + b) is equal to the fixed amount and the slope (m in y = mx + b) is equal to the variable or "per something" amount. In this case, the situation can be modeled 210 = 45h + 30, so h = 4. See equation building.

3) C) Calculate how many gallons each vehicle needs to travel 1,008 miles. If vehicle A gets 14 miles/gallon, it needs 1008/14 = 72 gallons. If vehicle B gets 36 miles per gallon, it needs 1008/36 = 28 gallons. 72 – 28 gives you the answer, 44. You know you need to divide miles by miles/gallon to get the gallons needed by each car because it causes the miles to cancel, leaving you with gallons as your unit. If the units are correct, you likely set the problem up right.

4) J) Combine like terms. The t^2 and the $-82t^2$ combine to give you $-81t^2$. The -59t and the 60t combine to give you t, so the answer is J.

5) C) If BCDE is a square and DC is 6 units in length, then DE, CB, and EB are also each 6 units in length. Because ABE is equilateral, BA and AE are also each 6 units in length. Therefore, the perimeter of ABCDE is composed of 5 segments each 6 units in length. 5 • 6 gives you a perimeter of 30 units. See plane geometry.

6) J) Here you must FOIL. Expand this expression by multiplying the First terms of each, the Outer terms of each, the Inner terms of each, and the Last terms of each. 4z • z + 4z • -2 + 3 • z + 3 • -2 = $4z^2$ – 8z + 3z – 6. Combine like terms to get $4z^2$ – 5z – 6. See quadratics.

7) C) Construct an equation using the first part of the sentence: .4x = 8 x = 8/.4 = 20. Then, find 15% of 20. .15 • 20 = 3. See percentages and equation building.

8) H) If the sum of the listed integers is 447, then 6x + 3 = 447 6x = 444 x= 74.

9) D) Deal with the x-coordinates and the y-coordinates separately on midpoint problems. If the x-coordinate of one endpoint is 7 and the x-coordinate of the midpoint is 5, then the x-coordinate of the other endpoint must be two units to the other side of 5, so 3. If the y-coordinate of one endpoint is 3 and the y-coordinate of the midpoint is 4, then the y-coordinate of the other endpoint must be one unit to the other side of 4, so 5. Therefore, the coordinates of endpoint A are (3, 5). If you were aware of your answer choices, you would have been done as soon as you got the x-coordinate, since only one answer has an x-coordinate of 3. The above method is much faster than plugging the known values into the midpoint formula to solve for the other endpoint, although that method would work as well, as long as you put the pieces in the right places. See coordinate geometry.

10) F) One way to do this is to just draw in the points and see which one appears to make a rectangle with points A, B, and C. (10, -3) is the only one that looks right. A more mathematical approach is that AD must be the same length as BC and the same slope as BC since it is parallel to BC. From B to C is down 8 and to the right 6, so from A to D must be down 8 and to the right 6 as well. See coordinate geometry.

11) E) You can apply the rules of matrix multiplication if you know them, but knowing these rules are unnecessary to getting the problem correct since you just need to work with the situation. Multiply the total number of T-shirt A's by the cost per T-shirt A, the total number of T-shirt B's by the cost per T-shirt B, and the total number of T-shirt C's by the cost per T-shirt C. Then, add these values together. (120 + 100) • 5 + (200 + 50) • 10 + (150 + 100) • 15 = $7350.

12) J) $\angle$y = 180 – 72 = 108. $\angle$x = 180 – 57 = 123. The missing angle of the triangle is 180 – 72 – 57 = 51, so $\angle$z = 180 – 51 = 129. Therefore x + y + z = 108 + 123 + 129 = 360. The faster way of doing this problem is to know that an exterior angle to a triangle is equal to the sum of the two nonadjacent angles of the triangle. Therefore, the sum of the exterior angles to each of the three angles of a triangle is equal to twice the sum of the angles of the triangle. The sum of the angles of a triangle is always 180, so the sum of the three exterior angles to the triangle's three angles is always 180 • 2 = 360. The numbers given in the figure were never necessary, nor was finding the values of x, y, or z. See plane geometry.

13) A) Divide the number of people who chose Whitney by the total number of voters polled. 30/200 = .15 = 15%. See percentages.

14) H) Calculate the percent of people polled who favored Lue. 80/200 = .4 = 40%. Then find 40% of 10,000. .4 • 10,000 = 4,000. See percentages.

15) B) First, calculate the percent of people polled who favored Gomez. 40/200 = .2 = 20%. Then, find 20% of the total 360 degrees that make up a circle. .2 • 360 = 72. See percentages.

16) G) The best way to do this problem is visually. Remember that even though the directions indicate otherwise, all figures are drawn to scale. Draw a line segment from D to the midpoint of AB. This divides triangle ADB into two congruent triangles, each of which are also congruent to triangle ADE. Therefore, the area of triangle ADE is ½ that of triangle ADB. Another way to think about it is that the two triangles have the same height, but the base of triangle ADB is twice the base of triangle ADE. Therefore, it would result in triangle ADB having twice the area of triangle ADE when plugged into the area formula for a triangle A = ½ bh. See plane geometry.

17) E) Parallel lines have the same slope, and the slope of the given line is 2/3, since 2/3 is the m in y=mx + b (slope-intercept) form. See coordinate geometry.

18) H) If the ratio of the lengths of the pieces is 2:3, that means that the short one is 2/5 of the total and the long one is 3/5 of the total. Therefore, the length of the short one is 2/5 • 30 = 12 ft. If this equation is not intuitive once you know that the short one is 2/5 of the total (remember that "of" typically means multiply), you could set up a proportion: 2/5 = x/30. This proportion works because the units line up: short side/total = short side/total. The five in the denominator is found by adding the 2 and 3. In the problem, you were given part to part, so it was necessary to find the whole by adding the parts together. Always pay attention to whether you are given part to part or part to whole. See ratios and proportions.

19) C) To find "the smallest integer greater than $\sqrt{58}$," enter $\sqrt{58}$ on your calculator and round the decimal up to the next integer, 8. You could also do this quickly in your head by recognizing that the answer is equal to the square root of the next perfect square greater than 58. This number is 64, and $\sqrt{64}$ = 8.

20) G) If each wall is 10ft x 15ft, the area of each wall is 150 square feet. With four walls, that is a total area of 150 • 4 = 600 square feet. You must then subtract the window, which is 3 • 5 = 15 square feet, and the door, which is 3.5 • 7 = 24.5 square feet. Therefore, the total area that must be painted is 600 – 15 – 24.5 = 560.5 square feet. If each gallon of paint paints 300 to 350 square feet, two 1-gallon cans is the minimum number of cans Sergio needs to purchase.

21) A) To solve a quadratic, first get set it equal to 0 by getting everything on one side of the equation. x^2 + 2x – 8 = 0. To solve, graph it and find the x-intercepts, use the quadratic formula, or factor it, as I will demonstrate here. To factor, find two numbers that add to 2 and multiply to -8: 4 and -2. Therefore, the factored form of this quadratic is (x + 4) (x – 2) = 0, so x = -4 and x = 2. If you are not familiar with any method of solving a quadratic, you could have simply plugged in your answer choices into the initial equation and to see which ones work. Remember that this method can be used in any "solve for x" problem. See quadratics.

22) K) In this expression, the 3 in the numerator cancels with the 3 in the denominator. Subtract the exponents of the "a" terms to get 4 – 6 = -2. This would indicates that the a's all cancel except an a^2 left over in the denominator. Remember that this type of cancelling only works when everything in the numerator and denominator are multiplied together; in other words, it does not work if there is addition or subtraction going on in the numerator or denominator other than that which occurs within parentheses. See exponents and radicals.

23) E) The point could be located in quadrant II since points in quadrant II have a negative x-coordinate and a positive y-coordinate, or in quadrant IV since points in quadrant IV have a positive x-coordinate and a negative y-coordinate. Quadrant I doesn't work since its points have two positive coordinates, and quadrant III doesn't work since its points have two negative coordinates.

24) K) When modeling a linear situation, the y-intercept (b in y = mx + b) is equal to the fixed amount and the slope (m in y = mx + b) is equal to the variable or "per something" amount. Therefore, 5.25 is the slope and 1,400 is the y-intercept. Watch out because the correct answer, instead of being written as y = mx + b is written as y = b + mx. See equation building.

25) B) Similar figures have the same shape but not necessarily the same size. Therefore, their corresponding parts are in proportion. So set up a proportion. Shortest side/ perimeter = shortest side/perimeter, so 3/x = 7.5/35. Cross multiply to get 7.5x = 105 x=14. If you didn't know how to do this problem, at least take an educated guess using the figure, since all figures are drawn to scale despite the directions at the beginning of the test saying they are not. Just by looking carefully at the figure, you could probably get it down to answer choice A or answer choice B. See ratios and proportions.

26) G) You could do this problem by cross-multiplying, but doing so would involve unnecessary work. Instead, recognize that the numerators of the fraction are the same. Therefore, for the fractions to be equal, their denominators must be equal, so a$\sqrt{7}$= 7. Therefore a =7/$\sqrt{7}$= $\sqrt{7}$.

27) C) The best way to do this problem is to set up two equations. When modeling a linear situation, the y-intercept (b in y = mx + b) is equal to the fixed amount and the slope (m in y = mx + b) is equal to the variable or "per something" amount. The equation for the first balloon is h = 70 – 6s and for the second is h = 10 + 15s. Because we want to know when h will be equal for each balloon, you can set the right sides of the equations equal to each other via substitution. 70 – 6s = 10 + 15s 60 = 21s s = 2.9. Another way of doing this problem quickly is to realize that the balloons start out 60 meters apart and are converging at a rate of 21 meters per second, which indicates that they will be at the same height in 60/21 = 2.9 seconds. This method is quicker, but you have to see it.

Often, understanding a problem in terms of the situation eliminates the need to set up equations. See equation building.

28) J) In counting problems, you should consider how many events are occurring and how many ways each event can occur. Here you have 3 events (a road, bike path, and trail are each chosen). It might be wise at this point to write out slots for each event with multiplication symbols in between them: _•_•_. Then, populate these slots with how many ways each event can occur. Here, the first event is a road is chosen (4 ways this event can occur), the second event is a bike path is chosen (2 ways this event can occur), and the third event is a trail is chosen (6 ways this event can occur). So 4 • 2 • 6 = 48. See counting and probability.

29) E) Cube B has an edge length of 4 inches, so its volume is 4^3= 64 cubic inches.

30) G) In this problem, you simply need to plug the numbers into the correct places in the equation and solve for A, the current value. r = .04 (four percent is not r = 4, which is the error you made if you chose K), n = 5, and P = 10,000, so the equation is A = 10,000$(1.04)^5$. Calculate A using your calculator.

31) D) You simply need to plug the radius and height of the cylinder into the given equation. The height is 20 and the radius is 20/2 = 10. 2π • 10^2 + 2π • 10 • 20 = 200π + 400π = 600π. If you chose E, you likely used 20 (the circle's diameter) as the radius, forgetting to divide it by 2.

32) H) $f(g(x)) = f(x^2 – 2)$. This is the case because $g(x) = x^2 – 2$, so $x^2 – 2$ can be substituted for g(x). $f(x^2 – 2) = 4(x^2 – 2) + 1 = 4x^2 – 7$. See function notation.

33) B) Multiply the "number of goals in a match" by the "number of matches with this total." Do this for every number of goals and add these numbers together to get the total number of goals scored in all 43 matches. Then divide this number by the total number of matches, 43, to get the average number of goals per match. 0 • 4 + 1 • 10 + 2 • 5 + 3 • 9 + 4 • 7 + 5 • 5 + 6 • 1 + 7 • 2 = 120. 120 goals / 43 games = 2.8 goals per game. See averages.

34) H) Because a and b are parallel, all the acute angles formed by the transversal c and either a or b are equivalent. These angles (1, 2, 9, and 10) are all supplementary to x since 1 forms a linear pair with x and 1, 2, 9, and 10 are all equivalent. None of the angles formed by the transversal d and a or b are equal to x because d is not parallel to c. See plane geometry.

35) E) The exponent outside the parentheses must be distributed to both terms in the parentheses. Therefore, you have $3^3 • (x^3)^3 = 27x^9$. See exponents and radicals.

36) F) When solving inequalities algebraically, remember to flip the inequality sign when multiplying or dividing both sides by a negative number. $-4x > 24$ so $x < -6$. If you answered G, you forgot to flip the sign. See inequalities and the number line.

37) C) The slope of the line connecting the center of the circle to the original P is (6 – 3)/(6 – 2) = 3/4. Because P is rotated 90 degrees clockwise, the new P would be the endpoint of a radius that is perpendicular to the slope 3/4. Because perpendicular lines have negative reciprocal slopes, this perpendicular slope is -4/3. To find the new location of point P, move down 4 units and right 3 units from the center: (2 + 3, 3 – 4) = (5, -1). If you didn't see how to do this problem, you could have gotten it down to answer choices C and D just by looking carefully at the diagram. See coordinate geometry.

38) K) Sin = opposite/hypotenuse, so sin M = KL/KM. Find KL using the Pythagorean Theorem. $10^2 + x^2 = 12^2$ $x^2 = 44$ $x = \sqrt{44}$. Therefore, sin M = $\sqrt{44}/12$. See basic trigonometry.

39) B) If BD bisects $\angle ABE$, then $\angle$ABD = $\angle$DBE. If BE bisects $\angle$CBD, then $\angle$DBE = $\angle$CBE. Therefore, $\angle$ABD = $\angle$DBE = $\angle$CBE. Since these three angles are equal and add to 180 (sine together they form a straight line), each one must be 180/3 = 60 degrees. If you didn't know how to do this problem, you could have visually estimated it to get the correct answer because the answer choices are so far apart. See plane geometry.

40) H) To find the hydrogen molecules per cubic centimeter, divide the hydrogen molecules by the cubic centimeters. (Remember that "per" means divide.) $(8 \bullet 10^{12}) / (4 \bullet 10^{4})$. You could either calculate this by hand or on your calculator. If you are doing it by hand, divide 8 by 4. Then to get the exponent of the 10, subtract 4 from12 since the bases are being divided. See exponents and radicals.

41) B) Although this problem looks difficult, it simply involves plugging numbers into the equation for the law of cosines, which is given to you in the note. First, find the measure of the angle made by connecting points A and B to the center: 300 – 170 = 130 degrees. At this point, the answer should be obvious; instead of painstakingly plugging in every piece, recognize that only one answer features an angle of 130 degrees. Based on the equation in the note, it is clear that you must use the angle opposite c, the side you are solving for.

42) J) "Half way between" implies that you must average 1/5 and 1/3, so add them together and divide by 2. Do this on your calculator to avoid having to find a common denominator. Then use the "frac" function in the "math" menu (on a TI calculator) to convert the answer to a fraction. Watch out for 1/4, as this is an easy trap to fall for if you aren't thinking carefully. See averages.

43) B) Label the point where the diagonals intersect E. By parallel line properties (alternating interior angles), $\angle$DBA = 25. Because the trapezoid is isosceles, it is symmetrical, so $\angle$CAB = 25 as well. Because the angles in a triangle add to 180, $\angle$AEB = 130. By supplementary angles, $\angle$BEC = 180 – 130 = 50. Because the angles of a triangle add to 180, $\angle$DBC = 180 – 35 – 50 = 95. If you didn't see how to do this problem, you could have eyeballed it using the figure. $\angle$DBC appears to be just a hair over 90. See plane geometry.

44) G) If the area of the larger square is 50, its side length is $\sqrt{50}$. If the area of the smaller square is 18, its side length is $\sqrt{18}$. Therefore, x = $\sqrt{50} - \sqrt{18}$. Do this on your calculator, and then use your calculator to see which answer choice represents an equivalent decimal. Alternatively, you can do this by hand. $\sqrt{50}$ = 5$\sqrt{2}$ and $\sqrt{18}$ = 3$\sqrt{2}$, so $\sqrt{50} - \sqrt{18}$ = 5$\sqrt{2}$ – 3$\sqrt{2}$ = 2$\sqrt{2}$. See exponents and radicals.

45) E) A rational number is defined as any number that can be written as the ratio of two integers (one integer divided by another). Another way of saying the same thing is that rational numbers include integers, terminating decimals, and repeating decimals. Numbers like π, e, and $\sqrt{5}$ are irrational. $\sqrt{64/49}$ is the only rational number of the answer choices because it simplifies to 8/7. See exponents and radicals.

46) K) If a $<$ b, then a – b is a negative number. Therefore, taking the absolute value of (a – b) would have the effect of changing its sign (making it positive). Multiplying (a – b) by -1 would have this same effect. See absolute values.

47) A) Set up an average equation. (Average of set) = (sum of items in set) / (number of items in set). Here, you know the average must be 80, and the number of tests is 6 (5 known tests and one unknown – the 8 tests total is irrelevant because he wants his average to be 80 after 6 tests). To find the sum of his scores, treat the 5 tests that average 78 as though they are each 78's, so 5 • 78. Then call the unknown x. So 80 = (5 • 78 + x)/6. Then use algebra to solve for x. See averages.

48) F) If you know what a modulus of a complex number is, by definition, its distance from the origin. In this case, z_1 is clearly the furthest point from the origin. If you have not learned about the concept of a modulus, enough is explained in the question for you to figure it out. Here, you need the largest value for $\sqrt{a^2 + b^2}$ (look like the distance formula?), with a the real coordinate and b the coefficient of the imaginary coordinate. Therefore, you need the real and imaginary numbers with the highest absolute values (the greatest distances from 0). This point is z_1.

49) C) Rewrite both sides of the equation using a common base. $(2^3)^{2x+1} = (2^2)^{1-x}$ When raising a power to a power, multiply the exponents, so $2^{6x+3} = 2^{2-2x}$. Therefore, 6x + 3 = 2 – 2x, so x = -1/8. See exponents and radicals.

50) F) Remember that f(x) refers to a y-value and the x refers to the x-value at that y-value. In other words, f(x) corresponds with the ordered pair (x, f(x)). Therefore, f(-x) refers to changing the sign of the x-value, which would reflect the function across the y-axis. -f(x) refers to changing the sign of the y-value, which would reflect the function across the x-axis. Therefore, this function is even, since its reflection across the y-axis is the same as the original function (it has y-axis symmetry). Another way you can think about why f(x) = f(-x) for this function is that the negative of every x-value produces the same y-value as the positive of that x-value. See function notation.

51) D) Probability is equal to the number of desired outcomes divided by the number of possible outcomes, so in this case it is the number of numbers from 100 through 999 that contain one or more zeros divided by the total number of numbers from 100 through 999. Figuring out the total number of numbers from 100 through 999 is easy: 999 – 100 + 1 = 900 (the one is added because both 100 and 999 are to be included). Finding how many of these numbers contain at least one zero is a tricky counting problem. In counting problems, you should consider how many events are occurring and how many ways each event can occur. Here you have 3 events (the hundreds, tens, and units digit are each chosen). It might be wise at this point to write out slots for each event with multiplication symbols in between them: _•_•_. Then, populate these slots with how many ways each event can occur. Here, it makes sense to do three separate counting problems: one for when the tens digit only is zero, one for when the units digit only is zero, and one for when the tens and units digits are both zero (the hundreds digit is never zero for the numbers we are working with). When the tens digit only is zero, there are 9 options for the hundreds digit (1-9), one option for the tens digit (0), and 9 options for the units digit (1-9), so 9 • 1 • 9 = 81. When the units digit only is zero, there are 9 options for the hundreds digit (1-9), 9 options for the tens digit (1-9), and 1 options for the units digit (0), so 9 • 9 • 1 = 81. When the tens and units digits are both zero, there are 9 options for the hundreds digit (1-9), one option for the tens digit (0), and one option for the units digit (0), so 9 • 1 • 1= 9. Therefore, the total number of numbers that have zero as at least one digit is 81 + 81 + 9 = 171. Divide 171 by 900 to get the probability of choosing a number with 0 as at least one digit. See counting and probability.

52) F) If $\angle$b = $\angle$a, then line r has a slope that is the negative version of the slope of line q. The equation of line q is y = 2x + 1, so its slope is 2. Therefore, the slope of line r is -2. See coordinate geometry.

53) D) Tan^{-1} a/b refers to the angle that has a tangent (opposite/hypotenuse) of a/b. This angle would be the bottom right angle on the diagram. The cosine (adjacent/hypotenuse) of this angle is b/$\sqrt{\mathrm{a}^2 + \mathrm{b}^2}$. See basic trigonometry.

54) J) The radius of the circular region of coverage is 52 miles, so its area is π • 52^2 = 2704π ≈ 8,500 square miles. See circles.

55) E) Standard form for a circle in the coordinate plane is $(x-h)^2 + (y-k)^2 = r^2$ where r is the radius and (h, k) is the center point. Have this equation memorized, as it comes up frequently. Here, the center point is the origin, so both h and k equal 0, and the radius is 52. See coordinate geometry.

56) G) This is an overlap problem. If the transmitters are 100 miles apart, and one transmits for 52 miles and the other for 60 miles, their transmission overlaps for 52 + 60 – 100 = 12 miles.

57) E) Simply observe the graph to determine the x-values for which the inequality is true. In other words, for which x-values does y = $(x-1)^4$ have a lesser y-value than that of y = x – 1. Observing the graph reveals that this is the case between x = 1 and x = 2, so $1 < x < 2$.

58) F) This problem is extremely abstract to solve in any way other than using your own numbers, so using your own numbers is the best way to go. Let's use 43 for x. This means that t = 4 and u = 3. If x = 43, then y = 34, so x – y = 9. Now you must substitute 4 for t and 3 for u in all the answer choices to see which answer or answers give you 9, the value of x – y. Answer choice F is the only one that equals 9, so it is your answer. Be careful, though; even if F is the first one you tried, and it works, you still must try all the other answer choices. It is possible that more than one answer will work coincidentally. If this happens, you will have to try again with different numbers. For this reason, you can only use process of elimination when using this technique.

59) A) CB is a base of length 5 – 1 = 4 (because the y-values of C and B are the same, they can be disregarded in calculating this length). Because the base is perfectly horizontal, the height must be perfectly vertical: the change in y from A to the line BC, so h = 5 – 3 = 2. The area of a triangle is ½ bh, so ½ 4 • 2 = 4. See plane geometry.

60) F) Simply plug the numbers into the equation: 200 = a / (1 – .15)   a = 200 • .85 = 170. Therefore, the first term is 170. To find the second term, multiply the first term by the common ratio: 170 • .15 = 25.5. See series and sequences.